

Non-Standard Time Reversal and Transverse Single-Spin Asymmetries

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Abstract

A system of quarks interacting with chiral fields is shown to provide a physical realization of a “non-standard” time reversal for particle multiplets which mixes the multiplet components. We argue that, if the internal structure of the nucleon is governed by a chiral dynamics, the so-called T -odd quark distribution functions are not forbidden by time-reversal invariance and hence might be non vanishing. This agrees with some other recent results. From a phenomenological point of view, this would give rise to single-spin asymmetries in inclusive processes involving a transversely polarized nucleon: in particular, in pion lepto- and hadro-production and in Drell-Yan processes.

1. Time-Reversal (TR) invariance is a powerful constraint on many physical processes. In the context of hadronic physics, for instance, it limits the admissible forms of structure functions, quark distributions, form factors, decay observables, *etc.* In this paper we shall present a physical realization of a “non-standard” time reversal for particle multiplets proposed by Weinberg [1] and characterized by a mixing of the multiplet components. The physical system that will be considered is an isospin doublet of quarks interacting with time-independent chiral fields (π ’s and σ ’s). We shall show that this system of quarks and mesons is indeed invariant under the non-standard TR. As an interesting consequence, if the nucleon has a chiral dynamics and time-reversal invariance is implemented according to Weinberg’s inversion operator for quark multiplets, some quark distribution functions that might appear to be time-reversal odd, do not actually conflict with TR invariance, and might therefore be non vanishing (a preliminary account of the ideas presented here was given in [2]). Our conclusions agree with a recent paper by Collins [3], who also finds that time-reversal invariance – when taking into account the path-ordered exponentiation of the gluon field in the operator definition of parton densities – cannot forbid new kinds of spin and \mathbf{k}_\perp dependent distribution functions. This important result opens the way to a rich and interesting phenomenology of transverse single-spin asymmetries.

2. Acting on a momentum and spin eigenstate $|\mathbf{p}, j_3\rangle$, the TR operator T yields

$$T |\mathbf{p}, j_3\rangle = (-1)^{j-j_3} |-\mathbf{p}, -j_3\rangle, \quad (1)$$

where j is the particle’s spin, j_3 its third component, and an irrelevant phase has been omitted. Recall also that T maps “in” states into “out” states: $T : |\text{in}\rangle \rightarrow |\text{out}\rangle$. Consider now a multiplet of particles labeled by some internal quantum number a . In the standard realization of TR, the T operator is taken to be diagonal in a :

$$T |\mathbf{p}, j_3, a\rangle = (-1)^{j-j_3} |-\mathbf{p}, -j_3, a\rangle. \quad (2)$$

In his Quantum Field Theory book [1] Weinberg has considered a more general possibility, namely that T may mix the multiplet components (an idea originally due to Wigner [4]). Thus a non-diagonal finite matrix \mathcal{T}_{ab}

appears in (2), which becomes

$$T |\mathbf{p}, j_3, a\rangle = (-1)^{j-j_3} \sum_b \mathcal{T}_{ab} |-\mathbf{p}, -j_3, b\rangle. \quad (3)$$

Since T is antiunitary, \mathcal{T} must be unitary. Weinberg has proven that the matrix \mathcal{T} can be made block-diagonal by a unitary transformation, with the blocks being either simple phases, or at most 2×2 matrices of the form

$$\begin{pmatrix} 0 & e^{i\phi/2} \\ e^{-i\phi/2} & 0 \end{pmatrix}, \quad (4)$$

where ϕ is a real number.

Weinberg's "non-standard" time reversal may indeed be realized in quark physics, as we are now going to illustrate (hereafter we refer to (2) and (3) as to the "standard" and "non-standard" TR, respectively).

Let us consider a $SU(2)$ chiral lagrangian describing the interaction of a quark field ψ with two chiral fields σ and $\boldsymbol{\pi}$

$$\begin{aligned} \mathcal{L} &= i \bar{\psi} \gamma^\mu \partial_\mu \psi - g \bar{\psi} (\sigma + i \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}) \psi \\ &+ \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \boldsymbol{\pi})^2 - U(\sigma, \boldsymbol{\pi}), \end{aligned} \quad (5)$$

where $U(\sigma, \boldsymbol{\pi})$ is a mexican-hat potential. Lagrangians of the type (5) are widely used to describe the structure of hadrons [5]. For simplicity, we shall assume that the chiral fields represent a time-independent background (this is a common assumption in many chiral models of the nucleon, see *e.g.*, [6]). The field equations obtained from the lagrangian (5), in the mean field approximation, are [6]

$$[i \gamma^\mu \partial_\mu - g (\sigma + i \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})] \psi(x) = 0, \quad (6)$$

$$-\nabla^2 \boldsymbol{\pi} + i N_c g \bar{\psi}(x) \gamma_5 \boldsymbol{\tau} \psi(x) + \frac{\partial U}{\partial \boldsymbol{\pi}} = 0, \quad (7)$$

$$-\nabla^2 \sigma + N_c g \bar{\psi}(x) \psi(x) + \frac{\partial U}{\partial \sigma} = 0. \quad (8)$$

Under TR the Dirac equation becomes (the superscript "t" denotes transposed)

$$[i \gamma^\mu \partial_\mu - g (\sigma - i \gamma_5 \boldsymbol{\tau}^t \cdot \boldsymbol{\pi}')] \gamma_5 \mathcal{C} \psi^*(\tilde{x}) = 0, \quad (9)$$

where $\tilde{x} = (-x_0, \mathbf{x})$, $\mathcal{C} = i\gamma_2\gamma_0$ and $\boldsymbol{\pi}'$ is the time-reversed pionic field. Were pions absent, the time-reversed quark field solution would be, as usual, $\gamma_5 \mathcal{C} \psi^*(\tilde{x})$. But in (9) the term containing $\boldsymbol{\pi}$ has changed sign and we need to specify how the pion field transforms under TR in order to get the time-reversed quark solution. Under TR the equation for the pion field becomes

$$-\nabla^2 \boldsymbol{\pi} - i N_c g \bar{\psi}'(x) \gamma_5 \boldsymbol{\tau}^t \psi'(x) + \frac{\partial U}{\partial \boldsymbol{\pi}'} = 0, \quad (10)$$

where primes denote time-reversed fields. By inspecting eqs. (9) and (10) we recognize the existence of two possible realizations of TR operator. The first one amounts to keeping the standard TR for the quarks and reversing the sign of the x and z components of the pionic field under TR. This realization of TR is generally used in systems in which pions are emitted or absorbed by a fermion in a perturbative way. The second one consists in leaving the pion field unchanged and performing an isospin rotation on the quark field. Since $\tau_2(-\boldsymbol{\tau}^t)\tau_2 = \boldsymbol{\tau}$, the time-reversed quark field is (ab are isospin indices)

$$T\psi_a(x)T^{-1} = -(\tau_2)_{ab} \gamma_5 \mathcal{C} \psi_b^*(\tilde{x}), \quad (11)$$

to be compared with the *standard* action of TR,

$$T\psi_a(x)T^{-1} = -\gamma_5 \mathcal{C} \psi_a^*(\tilde{x}). \quad (12)$$

The *non-standard* TR (11) exhibits a unitary isospin rotation τ_2 , which is exactly of the form (4) indicated by Weinberg, with $\phi = -\pi$. Thus the system of quarks and chiral fields governed by (5) exemplifies Weinberg's unconventional representation of TR. The existence of various possible definitions of time reversal operator is due to the degeneracy of the ground state of our model. It is also important to notice that none of the degenerate ground states is a flavor eigenstate [7]. From this viewpoint, the use of the non-standard TR operator is very natural.

Note that, if we generalize the lagrangian (5) to $SU(3)$, it is straightforward to show that there is no unitary irreducible 3×3 matrix that gives the time-reversed solution. This agrees with Weinberg's conclusion that non-standard TR can mix at most two components of the particle multiplet.

3. In most practical calculations of nucleon structure, the so-called hedgehog form of the pion field, $\boldsymbol{\pi} = \hat{\mathbf{r}} \phi(r)$, is adopted. In this case one is able to compute explicitly the ground-state quark configuration, which turns out to have the spin-isospin structure [7]

$$|h\rangle = \frac{1}{\sqrt{2}} (|u-\rangle - |d+\rangle) . \quad (13)$$

This is not a state of fixed isospin, but rather an eigenstate (with zero eigenvalue) of the so-called grand-spin $\mathbf{G} = \mathbf{J} + \mathbf{I}$ (spin + isospin). It is immediate to check that the hedgehog solution (13) is indeed invariant under the non-standard TR.

It is important to notice that the nucleon state built up from the chiral lagrangian (5) satisfies the usual TR properties. In the baryon rest frame, starting from the mean-field solution which is a superposition of nucleon and delta, we project out a state with definite spin and isospin by means of [8]

$$P_{J_3 I_3}^J \equiv (-1)^{J+I_3} \frac{2J+1}{8\pi^2} \int d^3\Omega \mathcal{D}_{J_3, -I_3}^{J*}(\Omega) R(\Omega) , \quad (14)$$

where $\mathcal{D}_{J_3, -I_3}^J(\Omega)$ is the familiar Wigner function and $R(\Omega)$ is the rotation operator. Due to the symmetry of the hedgehog (which has grand-spin zero), the rotation can be performed either on spin or on isospin. If we choose to perform a spin rotation, a spin-isospin baryon eigenstate (*e.g.*, a nucleon) is obtained as

$$|J, J_3, I_3\rangle = P_{J_3 I_3}^J |H\rangle, \quad (15)$$

where $|H\rangle$ is the mean-field solution, made of three quarks in the configuration (13) surrounded by a coherent state of pions. We now want to check that the TR operator for the baryon state defined by (15) is the usual one, that is $T = K \Theta(\sigma)$, where K is the operator of complex conjugation and $\Theta(\sigma) = -i\sigma_2$. Under T , the projector transforms as

$$\begin{aligned} T P_{J_3 I_3}^J T^{-1} &= (-1)^{J_3 - I_3} P_{-J_3, -I_3}^J \\ &= (-1)^{J - J_3} P_{-J_3, I_3}^J (-i\tau_2) . \end{aligned} \quad (16)$$

Since the hedgehog state is invariant under the combined action of $\Theta(\sigma) \Theta(\tau)$, applying (16) to (15) gives

$$T P_{J_3 I_3}^J |H\rangle = (-1)^{J - J_3} P_{-J_3, I_3}^J |H\rangle , \quad (17)$$

which is the standard time-reversal transformation, as anticipated.

4. Let us now explore the implications of the non-standard TR on the spin structure of hadrons. Although, as we have just seen, the TR operator defined in (11) does not affect the time-reversal properties of the nucleon as a whole, it does have important consequences on the internal quark dynamics. In particular, we shall see that the so-called “ T -odd” quark distribution functions, introduced by some authors [9, 10, 11, 12] to account for the single-spin asymmetries experimentally observed in transversely polarised pion hadroproduction [13], are actually allowed by TR invariance, if the TR operator acting on the quark fields is the one given in (11).

When used to constrain the general form of the quark-quark correlation matrix Φ in nucleons, which incorporates all quark distribution functions, TR invariance, implemented in the standard way via the operator (2), forbids correlations of the form

$$\varepsilon^{\mu\nu\rho\sigma} P_\nu k_{\perp\rho} S_{\perp\sigma}, \quad (18)$$

$$\varepsilon^{\mu\nu\rho\sigma} P_\rho k_{\perp\sigma}, \quad (19)$$

where P, S are the proton’s four-momentum and spin, respectively (\perp denoting the transverse components of S), and \mathbf{k}_\perp is the transverse momentum of quarks. The leading-twist T -odd quark distributions arising from the terms in Φ of the form (18) and (19) are called $f_{1T}^\perp(x, \mathbf{k}_\perp^2)$ and $h_1^\perp(x, \mathbf{k}_\perp^2)$, respectively. The former is related to the number density of unpolarised quarks in a transversely polarised nucleon; the latter measures the transverse polarisation of quarks in an unpolarised hadron. If we call $\mathcal{P}_{q/p}(x, \mathbf{k}_\perp)$ the probability to find a quark with momentum fraction x and transverse momentum \mathbf{k}_\perp in the proton, we have [14]

$$\begin{aligned} & \mathcal{P}_{q/p^\uparrow}(x, \mathbf{k}_\perp) - \mathcal{P}_{q/p^\uparrow}(x, -\mathbf{k}_\perp) \\ &= -2 \frac{|\mathbf{k}_\perp|}{M} \sin(\phi_k - \phi_S) f_{1T}^\perp(x, \mathbf{k}_\perp^2) \\ &\equiv \sin(\phi_k - \phi_S) \Delta_0^T f(x, \mathbf{k}_\perp^2), \end{aligned} \quad (20)$$

$$\mathcal{P}_{q^\uparrow/p}(x, \mathbf{k}_\perp) - \mathcal{P}_{q^\downarrow/p}(x, \mathbf{k}_\perp)$$

$$\begin{aligned}
&= -\frac{|\mathbf{k}_\perp|}{M} \sin(\phi_k - \phi_S) h_1^\perp(x, \mathbf{k}_\perp^2) \\
&\equiv \sin(\phi_k - \phi_S) \Delta_T^0 f(x, \mathbf{k}_\perp^2),
\end{aligned} \tag{21}$$

where the arrows denote transverse polarisation states, and ϕ_k and ϕ_S are the azimuthal angles of \mathbf{k}_\perp and \mathbf{S}_\perp , respectively. The field-theoretical definitions of $\Delta_0^T f$ and $\Delta_T^0 f$ are ($|P, \pm\rangle$ are the momentum-helicity eigenstates of the proton and a is the flavor index)

$$\begin{aligned}
\Delta_0^T f(x, \mathbf{k}_\perp^2) &= \text{Im} \int \frac{dy^- d^2 \mathbf{y}_\perp}{2(2\pi)^3} e^{-ixP^+ y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\
&\times \langle P, - | \bar{\psi}_a(0, y^-, y_\perp) \gamma^+ \psi_a(0) | P, + \rangle,
\end{aligned} \tag{22}$$

$$\begin{aligned}
\Delta_T^0 f(x, \mathbf{k}_\perp^2) &= \int \frac{dy^- d^2 \mathbf{y}_\perp}{2(2\pi)^3} e^{-ixP^+ y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\
&\times \langle P | \bar{\psi}_a(0, y^-, y_\perp) i\sigma^{2+} \gamma_5 \psi_a(0) | P \rangle.
\end{aligned} \tag{23}$$

The distribution $\Delta_0^T f$ was first introduced by Sivers [9] and its phenomenological implications were investigated in [10, 11, 12]; in some of these papers $\Delta_0^T f$ is denoted by $\Delta^N f_{q/p\uparrow}$. The distribution $\Delta_T^0 f$ was studied by Boer and Mulders [11, 15]. Using the standard action of TR on quark fields, eq. (12), it is easy to show that the matrix elements in (22, 23), and the corresponding distributions, change sign under T ,

$$T : \Delta_0^T f_q \rightarrow -\Delta_0^T f_q. \tag{24}$$

Therefore, invariance under standard TR implies that these distributions must vanish, as – in a first paper – pointed out by Collins [16].

This conclusion would be inescapable if the quark fields were *free*. However, they are *interacting* fields and this renders the implementation of TR invariance rather subtle. If quark interactions are modelled by a chiral lagrangian like (5), the TR operator acts as in (11), and consequently TR transforms the u quark distribution into minus the d quark distribution, and viceversa:

$$T : \Delta_0^T f_u \rightarrow -\Delta_0^T f_d. \tag{25}$$

A similar relation holds for $\Delta_T^0 f$. Therefore, what TR invariance entails is not the vanishing of the u and d distributions separately, but of their isoscalar combination

$$\Delta_0^T f_u + \Delta_0^T f_d = 0. \quad (26)$$

A caveat is in order. We cannot take the relation (26) too literally. In fact, it arises from the assumptions we made about the quark dynamics (in particular, the time-independence of the chiral fields). Hence, we should not expect (26) to hold in general. What we do expect, however, is that non-perturbative quark dynamics realizes TR in a non-standard way, so that our conclusion that the distributions $\Delta_0^T f$ and $\Delta_T^0 f$ are *not* necessarily forbidden by TR invariance should be general and quite firm.

The same conclusion has been recently reached by Collins, who has reconsidered his proof of the vanishing of $\Delta_0^T f$ based on (22) and (24). The correct field-theoretical expressions of quark distributions contain link operators (*i.e.*, path-ordered exponentials of the gluon field, the so-called Wilson lines), which are usually omitted (being unity in the axial gauge), but have a non-trivial time-reversal behavior. It turns out that, under T , a future-pointing Wilson line is transformed into a past-pointing Wilson line, and hence the distribution $\Delta_0^T f$ does *not* simply change sign for time reversal. The conclusion (24) is therefore wrong and time reversal invariance, rather than constraining $\Delta_0^T f$ to zero, gives a relation between asymmetries probed in different processes [3].

We have already mentioned that the T -odd distribution functions have interesting phenomenological consequences. In particular, the non vanishing of $\Delta_0^T f$ could explain [10] the azimuthal asymmetries in pion hadro-production observed by the E704 experiment [13]. In order to justify the existence of T -odd distribution functions, their proponents [10, 11, 12] invoke initial-state effects which would produce non-trivial relative phases between the colliding hadrons, thus preventing implementation of the “naïve” time-reversal invariance via (1). The situation would be specular to that occurring in the fragmentation process of single-inclusive lepto-production, where T -odd fragmentation functions are made possible by non trivial *final-state interactions*. It is quite difficult, however, to figure out a physical mechanism giving rise to *initial-state interactions*, but still preserving the QCD factorization. A corol-

lary of this line of reasoning is that T -odd distributions should be observable only in reactions involving two initial hadrons (*i.e.*, in Drell-Yan processes, hadron production in proton-proton collisions, etc.). What we have shown here is that $\Delta_0^T f$ and $\Delta_T^0 f$ are *not* T -odd, if TR is implemented via (11). If our idea is correct, these distributions could be probed not only in pion *hadro*-production, but also in pion *lepto*-production. Focusing on $\Delta_0^T f$, we get a leading-twist contribution to the cross section for the semi-inclusive process $\ell p^\dagger \rightarrow \ell \pi X$, which reads (for a review of transversely polarised lepton production of hadrons see [14])

$$\begin{aligned} \frac{d\sigma}{dx dy dz d^2\mathbf{P}_\perp d\phi} &= \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \frac{1+(1-y)^2}{y} \\ &\times \sum_{q,\bar{q}} e_q^2 \Delta_0^T f_q(x, \mathbf{P}_\perp^2) D_q^\pi(z) \sin\phi, \end{aligned} \quad (27)$$

where $D_q^\pi(z)$ is the fragmentation function of quarks into pions, and ϕ is the angle between the direction of the transverse spin of the target and the momentum \mathbf{P}_\perp of the produced pion (in the γ^*-p c.m. frame). Some evidence of non-zero single-spin asymmetries in lepto-production has been reported by the SMC [17] and the HERMES Collaboration [18].

5. We have shown that a system of interacting quark and chiral fields provides a physical realization of a non-standard TR that mixes the components of an isospin multiplet. As a consequence, the T -odd distribution functions of the nucleon are not obliged to vanish and may generate single-spin asymmetries in lepto-production. Recently, Brodsky, Hwang and Schmidt [19] have proven that such asymmetries may arise at leading twist due to gluon exchange between the outgoing quark and the target spectator system. This contribution is not power-law suppressed in Q^2 , and behaves as $1/\mathbf{k}_\perp^2$. Collins [3] has pointed out that the mechanism proposed by Brodsky *et al.* is compatible with QCD factorization and can be ascribed to a transverse-spin asymmetry in the \mathbf{k}_\perp distribution of quarks, that is to a $\Delta_0^T f$ function. Although Collins argument for the non vanishing of $\Delta_0^T f$ is quite different from the one presented in this paper, it has an important point in common with it: both mechanisms show that quark interactions (in the form of gluon exponentials in one case, of chiral fields in the other case) deeply af-

fect the time-reversal behavior of distribution functions. It is also important to remark that distribution functions are usually computed using effective lagrangians, like the chiral model one, which are appropriate at a low momentum scale. The argument presented in [3] on the non-vanishing of T -odd distribution functions is based on QCD and involves the dynamics of the gluonic degrees of freedom. On the other hand, in model lagrangians, the effective degrees of freedom are different from the ones of QCD and, for instance, chiral degrees of freedom can take the place of gluonic ones. It is therefore important to show how the argument presented in [16] can be bypassed without making an explicit use of gluon dynamics. The result presented here goes along these lines, offering a realization of the idea discussed in [3] in terms of chiral degrees of freedom.

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